Gläser geschlossene Rb-Sr-Systeme während des Einschmelzens, und sind die Tektite von ähnlichem Ausgangsstoff abzuleiten, dann muß man jedoch Fraktionierung von Rb und Sr annehmen, um die Abweichung der Tektitenresultate von der "Isochrone" der Gläser zu erklären. Demnach ist es nicht möglich, genauere Folgerungen aus unseren Messungen hinsichtlich des Ursprungs der EK-Tektite zu gewin-

nen. Es ist jedoch gesichert, daß eine deutliche Ähnlichkeit zwischen den Rb-Sr-Beziehungen beider Glasarten besteht.

Wir danken Professor W. Gentner und Dr. E. T. C. Chao für die Überlassung des Probenmaterials und Diskussion einschlägiger Fragen. Die Arbeit wurde unterstützt durch die National Aeronautics and Space Administration (NGR-05-002-028) und eine Beihilfe der National Science Foundation.

Remarks on Lunar and Asteroidal Meteorites

K. SITTE

Max-Planck-Institut für Kernphysik, Heidelberg

(Z. Naturforschg. 21 a, 231-237 [1966]; received 23 December 1965)

Based on the experimental results of Gault et al., and on the orbit calculations of Arnold, the relative strength of the contributions from impacts of bodies in solar orbit on the asteroids and on the moon has been estimated. It is concluded that the asteroidal belt accounts for at least a considerable fraction of all stony meteorites, but lunar impacts occurring at a rate of about one in a few 10^5 years cannot be ruled out. Small bodies produced in asteroidal collisions which remain orbiting in the belt, can escape from it as a result of repeated "elastic" collisions by multiple scattering. Their life time in the belt is only about 1.4×10^5 years. Satisfactory values are found for the mass loss and for the replenishment of the debris in the asteroidal belt.

I. Basic Data and Assumptions

The dominant view on the origin of stony meteorites is that they are part of the debris ejected in the impact of bodies in solar orbits on other members of the planetary system. But how and where they are ejected has long been a subject of controversy. As to the first, the experimental work of GAULT et al. 1 has provided most of the answers. In the following we shall make use of their data on the velocity dependence of the cumulative mass of the ejecta, on the height or range distribution of the fragment mass in lunar impacts (showing, in particular, that at a projectile velocity of 28 km/sec the total mass of the escaping fragments is about ten times that of the incident body), and of the relation between the mass μ of all ejecta and the projectile mass m,

$$\mu = b \ m^{\beta} \tag{1}$$

with $\beta < 1$ because of the larger loss rate to irreversible processes at higher impact energies. The value $\beta = 3/4$ appears to give the best fit to their data. We shall also refer to their observation that the mass of

the largest fragment ejected is roughly proportional to the projectile mass.

As to the place of origin of stony meteorites, the asteroidal belt has been favoured for a long time. Recently, however, the possibility of lunar origin at least of certain types - the bronzites, enstatites and pigeonites - has again been seriously discussed (Zähringer², Wänke³). Their short exposure age can easily be explained on this basis but is hard to understand if a more distant source region is involved. For others like the hypersthenes and amphoterites whose exposure ages are in general significantly higher, this argument may not hold and the question must be raised whether these two groups of meteorites do not originate in different regions of space. In Section II of this note we shall attempt to derive an answer to this question from very general considerations. The problem will be treated in a crudely simplifying form only, but it is believed that the results do not depend critically on the approximations used.

Briefly, the argument can be put as follows: Although the nature of the projectiles impinging on



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung "Keine Bearbeitung") beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

D. GAULT, E. M. SHOEMAKER, and H. J. MOORE, NASA Technical Note D-1767, Washington, D.C. 1963.

² J. Zähringer, Meteoritika 1965, in press.

³ H. Wänke, Berichte über Meteoritenforschung No. 74, Max-Planck-Institut für Chemie, Mainz.

232 K. SITTE

the lunar surface cannot be ascertained as yet, there is scarcely any doubt that their orbits will also make them traverse the asteroidal belt. Consequently they will suffer collisions with the bodies of the belt as well, and eject fragments from them as from the moon. If we succeed in estimating the ratio of the masses μ_A and μ_L of the material collected by the earth as the result of the bombardment of asteroidal and lunar surface, we can answer our question. Clearly $\mu_A/\mu_L \ll 1$ would imply predominance of lunar origin; $\mu_A/\mu_L \gg 1$, conversely, would show that the asteroids are the major source region. But if $\mu_A \approx \mu_L$ one would be tempted to identify the types of meteorites of short exposure ages with lunar debris, and the others with material ejected from the asteroidal belt.

Two further assumptions have been made in these considerations. The first concerns the mass distribution of the — unspecified — projectiles for which a power law

$$f(m) dm = A \cdot m^{-\gamma} dm \quad (5/3 \le \gamma \le 2) \quad (2)$$

will be taken. A distribution of this kind has been observed for a variety of phenomena ranging from meteorids to asteroids, and as Hawkins ⁴ has pointed out, agrees with the comminution law valid for grinding processes. In numerical calculations we shall generally use the lower limit, 5/3, of the exponent which according to Piotrowski ⁵ is maintained in steady grinding.

Similarly, we shall assume a mass distribution

$$\Phi(M) dM = B \cdot M^{-\gamma} dM$$
 (3)

for the asteroidal bodies which occupy the belt. If the total mass of all asteroids $M_{\rm A} = 10^{-2}\,M_{\rm E}$, and the maximum mass of a single body $M_{\rm max} = 1.5 \times 10^{22}\,{\rm g}$ (corresponding to a radius of about 100 km), we obtain for the number of bodies of mass $\geq M$

$$N(M) = ((2-\gamma)/(\gamma-1)) \cdot (M_{A}/M_{max}) \cdot (M_{max}/M)^{\gamma-1} \quad (\gamma \neq 2). \quad (4)$$

We note for future reference that with $\gamma = 5/3$ the number of bodies of $M_{\rm min} \ge 1.5 \times 10^{10} \, {\rm g} \ (r_{\rm min} \ge 10 \, {\rm m})$ will be $N = 2 \times 10^{11}$.

For the shape of the asteroidal belt we use Arnold's "Astrid" model ⁶: a toroidal ring centered upon the sun, with a mean distance $R_0 = 2.75$ a. u. and a

radius $r_0 = 0.75$ a. u., as schematically shown in Fig. 1. Its volume V is, therefore, about 10^{41} cm³, and the average distance $\langle d \rangle$ between the bodies of $M \ge M_{\min}$ is of the order $(V/N)^{1/3} \approx 8 \times 10^9$ cm.

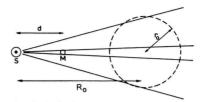


Fig. 1. Model of the asteroidal belt (ring of radius r_0 in distance R_0 from the sun). M shows the position of the moon in distance d=1 a.u.

In this model the asteroidal belt contains a large number of small bodies whose total mass, however, remains a small fraction of $M_{\rm A}$ if $\gamma < 2$. It will be shown in Section III that they will be removed from the belt by the process of multiple scattering in a time short compared with the average exposure age of meteorites.

II. Rates of Lunar and Asteroidal Meteorites

Since observable meteorites must have a certain minimum size, they can be produced only by projectiles of a mass exceeding a minimum value M_0 . For lunar meteorites, M_0 can be estimated in two independent ways: Firstly, no bronzites younger than about 100 000 years have been found, and altogether 10 of an age less than 2×10^6 y. We may, therefore, put the rate of incidence of meteoriteproducing bodies on the moon at about one in 105 v. and combine this information with the fact that on the lunar surface more than 300 000 craters of >1 km diameter have been observed. This represents the minimum number of craters formed since some will have been lost by erosion and obliteration. But the true number is probably not much higher because of the apparently small rate of erosion and of the small likelihood of obliteration. Thus the rate of formation of craters of diameters > 1 km will be of the order of one in 104 years or a little less, smaller than that of meteorite ejection by a factor, say, 20. Therefore, in view of the assumed size distribution (2) and of the near-proportionality between projectile radius and crater radius, meteorite-producing impacts will have excavated craters

⁴ G. S. Hawkins, Astronom. J. **65**, 318 [1960]; Nature **197**, 781 [1963].

⁵ S. Piotrowski, Acta Astron. (A) 5, 115 [1953].

⁶ J. R. Arnold, Astrophys. J. 141, 1536, 1548 [1965].

of a few km diameter. At an impact velocity around 30 km/sec, this corresponds to a projectile diameter of $\sim 100 \text{ m.} - \text{Secondly}$, we use the estimate of the total mass accretion on the earth due to meteoritic bodies, usually given as 1-10 tons/day. If, say, one ton of this comes from the moon, and if one-half of the material ejected from the lunar surface eventually reaches the earth (Arnold 6), then the total amount ejected in 10^5 years - i. e., by one event - is about 7×10^{13} g. Referring to Gault's results 1 mentioned above, we find that this requires a projectile mass of about 7×10^{12} g, or a projectile radius of about 70 m (iron) or 90 m (stone) - in almost embarrassingly good agreement with the first estimate 7.

If indeed lunar meteorites originate in impacts of bodies of different masses, that is in impacts of varying "efficiency", the distribution of their exposure ages will be different from that calculated by Arnold 6 which represents the survival probability of fragments from a single collision. But we can apply his results to evaluate the distribution which should be observed for strictly average behaviour of the projectiles. To do that, we shall use a smoothedout analytical representation of Arnold's data for the probability p(T) of an exposure age T before capture of a moon fragment by the earth:

$$p(T) = c \cdot T^{-\lambda} \tag{5}$$

which, as Fig. 2 shows, is a good approximation to the results of his Monte Carlo calculations.

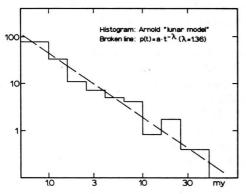


Fig. 2. Histogram of life times of lunar fragments according to Arnold 6, and the power law approximation (5) shown in the broken line.

Noting that fragments observable as meteorites must have masses exceeding a certain minimum value μ_0 , we find, first, for the total mass $\Delta\mu$ of all objects which after an impact of mass m become "potential meteorites"

$$\Delta \mu = \bar{\mu} \left[\left(m/m_0 \right)^{\beta} - 1 \right]. \tag{6}$$

Here we have made use of the various conditions stated above, and introduced the constant $\bar{\mu}$ for convenient presentation. — Next we note that for the average time intervals $T_{\rm m}$ and $T_{\rm 0}$ of impacts of masses m and $m_{\rm 0}$ the relation

$$(T_{\rm m}/T_{\rm 0}) = (m/m_{\rm 0})^{\gamma} \tag{7}$$

will hold. Finally, since no impact of mass m_0 has occurred within $T_0 \approx 10^5$ y, we shall assume that contributions from masses $\geq m$ have to be taken into account only for times $T \geq T_{\rm m}$ as defined in (7). This leads us to assign a weight factor w(T) to the probability of observing a certain life time T:

$$w(T) = (1/\alpha) \left[(T/T_0)^{\alpha/\gamma} - 1 \right] - (1/(\gamma - 1)) \left[1 - (T/T_0)^{-(\gamma - 1)/\gamma} \right]$$
(8)

where we have written for abbreviation $\alpha = \beta + 1 - \gamma$. From (5) and (8) we thus obtain a probability for observing a meteorite of exposure age T, originated in an impact of all likely masses m,

$$P(T) = C \cdot T^{-\lambda} \cdot w(T) . \tag{9}$$

P(T), calculated with $\gamma = 5/3$ and for $T_0 = 0.25$ and $T_0 = 1.0$ (all times in million years) is shown in Fig. 3 together with the histogram of exposure ages

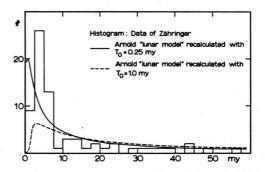


Fig. 3. Histogram of exposure ages of bronzites, enstatites and pigeonites according to Zähringer 2 , and calculated life times for two values of T_0 .

enormous tidal waves). Impacts producing glass or tektites, for which apparently projectiles of a diameter close to 1 km are required, would occur roughly once in 10⁷ years. Both rates seem entirely compatible with the scarce empirical evidence.

⁷ It is of interest to consider the consequences of this possible bombardment of the moon for the earth itself. Here the rate would be somewhat more than 10-times higher: An event of the type of the Cañon Diablo fall should be expected about once in 10,000 years (with the majority of falls, of course, into the ocean and producing only the temporary effect of

234 K. SITTE

of bronzites, enstatites and pigeonites reported by Zähringer². The choice of the parameter T_0 is rather arbitrary, as it is evident from the nature of the problem that perfect agreement can neither be achieved nor even aimed at. But the lower value of T_0 corresponds roughly to the expected average rate, while the higher was chosen to give a maximum of the distribution at about the age for which the observations show the highest frequency. Both calculated curves indicate that an unusually large-scale event is responsible for the abundant group of meteorites of ages around 4 my. However, the assumption of a bombardment by projectiles of a mass distribution according to (2) with an effective rate of one event in a few 105 y seems to be compatible with the experimental evidence.

We can now proceed to derive an estimate of the ratio μ_A/μ_L of the masses of meteoritic material collected by the earth from collisions in the asteroidal belt (μ_A) and on the lunar surface (μ_L) . This ratio will depend on three factors: the frequency of impacts N, their efficiency in ejecting debris η , and the probability of capture of the fragments by the earth p:

$$(\mu_{\rm A}/\mu_{\rm L}) = (N_{\rm A}/N_{\rm L}) \cdot (\eta_{\rm A}/\eta_{\rm L}) \cdot (p_{\rm A}/p_{\rm L}).$$
 (10)

Now N depends not only on the collision cross section, but also on a further geometrical factor, the solid angle subtended by the moon and by the asteroidal belt. The situation is schematically illustrated in Fig. 1. If we assume that the flux of projectiles is isotropic over this range of inclinations, we have

$$(N_{\rm A}/N_{\rm L}) = (\sigma_{\rm A}/\sigma_{\rm L}) \cdot (\omega_{\rm A}/\omega_{\rm L})$$

$$= (\sigma_{\rm A}/\sigma_{\rm L}) \cdot (d/R_0) \cdot (r_0/R_{\rm L})$$

$$= 2.35 \times 10^4 \cdot (\sigma_{\rm A}/\sigma_{\rm L})$$
(11)

writing $R_{\rm L}$ for the lunar radius. As to the cross sections, we can use the geometrical values π R^2 with negligible error, since even for the moon the effective radius of capture at impact velocities around 30 km/sec is still very close to $R_{\rm L}$. For the asteroidal belt, (3) allows us to calculate the cross section for collisions with bodies of fixed radius r, assuming that effective fragmentation occurs only if the target body has a radius $r' \gtrsim 10 \ r$:

$$\sigma_r = \pi R_{\text{max}}^2 \cdot K \cdot \ln(R_{\text{max}}/r')$$

$$(K = 3(2 - \gamma) (M_A/M_{\text{max}})) . \qquad (12)$$

The choice of r' is again suggested by Gault's results ¹ that in a high-speed impact the total mass of the ejecta exceeds the projectile mass by a factor

of the order 10^3 . Collisions with smaller bodies will, therefore, not contribute substantially to the fragmentation process. Also, we note that r' appears only in the logarithmic term so that its uncertainty does not affect the results appreciably.

The total cross section σ_{Λ} is now obtained by integration over r with the distribution function (2), and one finds (in the approximation

$$R_{\text{max}} \gg R_{\text{min}} \gg 10 R_0$$
,

the minimum radius of effective target asteroids

$$(\sigma_{\rm A}/\sigma_{\rm L}) = (R_{\rm max}/R_{\rm L})^2 \cdot K \cdot \ln(R_{\rm max}/R_{\rm min}). \quad (13)$$

This gives $(\sigma_A/\sigma_L) \approx 60$, and consequently

$$(N_{\rm A}/N_{\rm L})\approx 1.4\times 10^6$$
.

Turning now to the second factor in (10), we make use again of Gault's results 1 on the cumulative mass of ejecta as a function of the ejection velocity. His data show that after an impact at velocities around 30 km/sec on the moon, only a fraction between 10^{-2} and 10^{-3} of the total fragment mass can escape. Conditions are quite different in the asteroidal belt where even for the largest bodies the escape velocity is only about 140 m/sec. The major fraction of the fragments will be ejected at speeds exceeding this, and thus not be recaptured by the target body. This would suggest an "efficiency factor" of the order 100 favouring the asteroidal belt. However, this figure is too high for two reasons. Firstly, the average impact velocity at the distance of the asteroids is only about 2/3 of that on the moon, so that because of $\mu \propto E^{\beta}$ the efficiency ratio must be reduced by a factor ~2. Secondly, very slow fragments will not escape from the belt even if they are not immediately recaptured but predominantly end their lives in "inelastic" collisions with other asteroidal bodies. To survive those an ejection velocity of the order of 1 km/sec is needed which permits only about 3-5% of all fragments to escape. Taking into account both these factors, we shall adopt the value for the efficiency ratio

$$(\eta_{\rm A}/\eta_{\rm L}) \approx 5$$

which, however, may easily be in error by a factor ~ 3 both ways.

The estimate of the capture probability presents an even more difficult problem. Arnold's calculations ⁶ for the asteroidal belt show that 0.4% of all fragment orbits cross the orbit of Mars, and thus become candidates for capture by the earth. For the

probability of this event he derives a value of 1.4×10^{-4} — in the case of stony meteorites — because of their frequent destruction in further passage through the belt. But this should be taken as a lower limit because his model demands an improbably large cross section for the asteroids. His value of 1/2 for the capture probability of lunar fragments, on the other hand, is directly applicable to our problem. We shall, therefore, use

$$(p_{\rm A}/p_{\rm L})\gtrsim 1.1\times 10^{-6}$$
.

Combining these three factors we obtain finally

$$(\mu_{\rm A}/\mu_{\rm L}) \gtrsim 7.7$$
.

It is difficult to assess with confidence the uncertainties of these figures. We have probably overestimated the geometrical factor (ω_A/ω_L) , though not by orders of magnitude, and underestimated (p_A/p_L) . These possible systematic errors, and the uncertainty in (η_A/η_L) , permit us as a safe conclusion only the statement that the asteroidal belt cannot be considered as a negligible or minor source of meteorites. The results seem compatible with the assumption that the short-exposure age meteorites originate on the moon, and the long-exposure age types in the asteroidal belt [in which case we should expect $(\mu_A/\mu_L) \approx 2$, or with predominance of asteroidal origin. A more definite answer could only be obtained if the experimental study of high-speed impacts can be advanced to a state which would make possible a quantitative study also of the mass distribution of lunar ejecta, and possibly if the astronomical features of the model discussed here were introduced in a more accurate form.

Finally, a re-calculation of the exposure age distribution of the asteroidal meteorites, based on Arnold's "short-life" model and the weight functions derived above but evaluated with $T_0 = 10^4 \, \mathrm{y}$ (according to the larger cross section) gives the

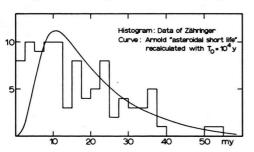


Fig. 4. Histogram of exposure ages of hypersthenes and amphoterites according to Zähringer 2 , and calculated life times for $T=10^4$ y.

data presented in Fig. 4. Again it is doubtful whether Arnold's model with its very high collision probability in the belt can be applied without correction. But it is even more questionable whether at present the physical and astronomical conditions could be defined reliably and accurately enough to warrant the effort of a more thorough calculation. Thus we can conclude this section only by stating that bombardment of asteroids and moon by projectiles of a mass distribution (2) will produce effects compatible with the general features of the experimental evidence.

III. Multiple Scattering in the Asteroidal Belt

It will now be shown that in a belt of the dimensions described in Section I, and populated according to (3), the life time of small asteroidal bodies is not determined by the process of actual impacts — the "inelastic collisions". Rather, the much more frequent distant, "elastic" collisions giving rise to small deflections in arbitrary direction, will eventually displace these bodies to distances larger than r_0 . In other words, small asteroidal bodies will diffuse out of the belt due to multiple scattering.

Since the bodies will rotate within the ring with a velocity v_0 of the order of $20 \, \mathrm{km/sec}$, we can assume that velocity differences of the order $v_0 \, (r_0/R_0)$ will occur. A reasonable estimate for the average relative velocity v of our "particles" will thus be $v=2 \, \mathrm{km/sec}$. Consider, now, the "collision" at an impact parameter b between two bodies of masses μ and $M, \, \mu \ll M$, moving at that relative velocity. The lighter particle will be deflected by an angle Θ_{s} ,

$$\Theta_{\rm s} = 2 G M/(b v^2) \tag{14}$$

where G is the gravitational constant. Averaging over the impact parameters and over the masses, one obtains for the mean square deflection in $\mathrm{d}x$ (with $M \ll M_{\mathrm{max}}$)

$$\langle \Theta_{s}^{2} \rangle = \mathrm{d}x \cdot 2 \,\pi \cdot ((2 - \gamma)/(3 - \gamma)) \cdot (M_{\mathrm{A}} \, M_{\mathrm{max}}/V)$$
$$\cdot (2 \, G/v^{2})^{2} \cdot \ln(b_{\mathrm{max}}/b_{\mathrm{min}}). \tag{15}$$

(Here we have simplified the calculation by neglecting the mass dependence of the logarithmic term. This does not introduce a serious error: note that $b_{\rm max} \! \approx \! \langle d \rangle / 2$, and $b_{\rm min} \! \approx \! r(M)$, so that

$$b_{\text{max}}/b_{\text{min}} = (1/2) \cdot [((\gamma - 1)/(2 - \gamma)) \cdot (V M_{\text{max}}^{2-\gamma}/M_{\text{A}}) \cdot (4 \pi \varrho/3)]^{1/s}$$
 (16)

236 K. SITTE

writing ϱ for the density of the asteroidal bodies. The dependence on M is slight, and negligible under the log.)

We shall further simplify the calculation by considering the diffusion problem in a cylinder instead of a toroidal ring. Though this is a good approximation only for $r_0 \ll R_0$, it will give us the right order of magnitude also in our case, and this is all we need for the present discussion. — The mean square displacement $\langle y^2 \rangle$ perpendicular to the direction of motion after traversal of a distance x becomes $\langle y^2 \rangle = \langle \Theta_s^2 \rangle \cdot x^3/12$, and the life time T of the particle, because of $x = v \cdot T$ and $\langle y^2 \rangle = r_0^2$,

$$T = (1/v) \cdot [(6/\pi) \cdot ((3-\gamma)/(2-\gamma)) \cdot (V r_0^2/M_A M_{\text{max}}) \cdot (v^2/2 G)^2/\ln(b_{\text{max}}/b_{\text{min}})]^{1/2}.$$
(17)

With the numerical values above, and with $\gamma = 5/3$, $M = M_{\text{min}} = 1.5 \times 10^{10} \text{ g}$, one obtains

$$T\!pprox\!4.3\! imes\!10^{12}\,\mathrm{sec}\!pprox\!1.4\! imes\!10^5\,\mathrm{y}$$
 .

It is easily seen that this surprisingly short life time in the asteroidal belt is not strongly affected by our choice of the exponent γ of the mass distribution law. Furthermore, the results should hold for all masses $\leq M$, provided only that $M \ll M_{\rm max}$ and that the conditions for applicability of the scattering formalism are satisfied.

Three conditions have to be met:

- (i) the mean free path λ_s between two "elastic" collisions must be small compared with r_0/Θ ;
- (ii) the time $T_{\rm e}$ between two "elastic" collisions must be small compared with the life time T in the belt;
- (iii) the time T_i between two "inelastic" collisions must be large compared with T.

Conditions (i) and (ii) ensure the multiple nature of the process, while (iii) shows its predominance over catastrophic collisions.

Turning to (i), we note that λ_s can be written

$$\lambda_{\rm s}(M) \approx V/N(M) \cdot \sigma$$
 (18)

where $\sigma = \pi b^2$ is the "cross section" at impact parameter b. Introducing (14) for Θ , condition (i) then leads to

$$N(M) \gg 8 G_{\varrho} V/(3 r_0 v^2)$$
 (19)

which with the numerical values chosen above gives

$$N(M) \gg 1.4 \cdot 10^{11}$$
.

The condition is thus satisfied for bodies of radii below 10 m, but not for larger ones.

As to (ii), we find that the average time between two elastic collisions, $t_{\rm e}\!\approx\!\langle d\rangle/v$, with the value of $\langle d\rangle$ derived in I is only about 4×10^4 sec, very short indeed compared with T. We also see that the distance covered in relative motion during T is $v\cdot T\!\approx\!8.3\times10^{17}$ cm, very large compared with $\langle d\rangle$. Multiple scattering can, therefore, be effective.

To check (iii) we shall assume that all collisions of bodies of mass M with others of the same or larger mass are "catastrophic", and evaluate the average collision time $T_i = \lambda_i/v$. The mean free path λ_i is determined by the total cross section Σ of the asteroidal bodies of mass $\geq M$,

$$\Sigma(M) = \int \pi R(M)^2 \cdot dN(M),$$

for which (3) gives

$$\Sigma = (\pi/3) \cdot (3/4 \pi \varrho)^{2/3} \cdot (M_{\text{A}}/M_{\text{max}})$$
$$\cdot M_{\text{max}}^{2/3} \cdot \ln(M_{\text{max}}/M) \quad (20)$$

if $\gamma = 5/3$, or

$$\Sigma = ((2 - \gamma)/(\gamma - 5/3)) \cdot \pi \cdot (3/4 \pi \varrho)^{2/3} \cdot (M_{\text{A}}/M_{\text{max}}) \cdot M_{\text{max}}^{2/3} \cdot (M_{\text{max}}/M)^{\gamma - 5/3}$$
(20 a)

if $\gamma = 5/3$. With the numerical values used above one obtains for the life time in years

$$T_i \approx 4 \times 10^{11} / \ln(M_{\text{max}}/M)$$
 $(\gamma = 5/3)$ (21)

0

$$T_i \approx ((2-\gamma)/(\gamma-5/3)) \times 1.25 \times 10^{10} \times (M/M_{\text{max}})^{\gamma-5/3} \quad (\gamma \neq 5/3). \quad (21 \text{ a})$$

Thus, condition (iii) is well satisfied for all masses if $\gamma=5/3$, and holds over a considerable range of masses even for larger γ . For instance, $\gamma=11/6$ — corresponding to a size distribution $r^{-3.5}\,\mathrm{d}r$ — gives $T_i=1.25\times 10^8\,\mathrm{y}$ for $M=1.5\times 10^{10}\,\mathrm{g}$, and still $T_i=4\times 10^6\,\mathrm{y}$ for $M=15\,\mathrm{g}$. We note the slight mass dependence of the life times; but only very small bodies will be eliminated predominantly by "inelastic" collisions. In the mass range of typical meteorites — say, from kg to tons — "multiple scattering" is both possible and predominant.

Let us still consider the mass loss of the belt due to scattering. If *all* bodies of masses below

$$M(\leq M_{\rm min} = 1.5 \times 10^{10} \text{ g})$$

are eliminated within T, one has

$$dM_{A}/dt = (1/T) \cdot \int_{0}^{M} M \, dN(M)$$
$$= (M/M_{\text{max}})^{2-\gamma} \cdot (M_{A}/T) . \qquad (22)$$

The fraction $\alpha=(M/M_{\rm max})^{2-\gamma}$ lost within T is about 4.65×10^{-5} if $M=0.1\,M_{\rm min}$, and 10^{-4} for $M=M_{\rm min}$. Thus even if the process had been equally effective during the entire history of the solar system, no catastrophic mass loss would have occurred. A life time of the belt in its present form $T_0=4\times 10^9$ y requires an initial mass $M_0\approx 4\,M_{\rm A}$ for the smaller α , and $M_0\approx 20\,M_{\rm A}$ for the higher. Recapture may, of course, reduce the mass loss.

The absolute mass loss according to (22) is about 5.5×10^7 tons/day. If the orbits of the escaping bodies resemble those of Arnold's calculations 6 mentioned above, the flux rates on earth will lie between 6.6×10^4 tons/day ("long life"), and 30 tons/day ("short life"). Recapture by the belt — its slow lateral expansion — may again reduce this figure with should be compared with the estimates of the total mass accretion of the earth, usually given as larger than 10^4 tons/day (e. g. Alexander et al. 8; Griebine 9). But it must be remembered that in our model "direct" ejection from inelastic collisions should contribute quite appreciably to the escape of fragments.

If small bodies diffuse out of the belt, the distribution must be replenished by debris from the collisions of the larger ones. To this, the impacts of bodies orbiting outside the belt — comets and/or heavy iron meteorites — may contribute substantially, but we can show that the asteroidal bodies themselves could also supply fragments in sufficient quantity. We shall estimate, in a first approximation, the fraction β of the colliding masses ejected as a result of the collisions by demanding

$$dM_{A}/dt = \beta \cdot \overline{m} \cdot \nu \tag{23}$$

where \overline{m} is the average mass of the bodies of mass above M_{\min} , and $\tilde{\nu}$ the frequency of their encounters. Since $\nu = N^2 v \sigma/V$, with $\sigma = \pi R^2$, one finds $\beta = 0.012$.

Thus, about one percent of the colliding masses must be ejected if *only* these processes contribute.

Lastly a remark concerning the exposure ages of asteroidal meteorites. In the model discussed here, they begin their lives as ejecta from large bodies, diffuse out of the belt in a time short compared with the ages generally observed, and spend the remainder of their lives in orbits outside the belt though probably crossing it in most of their revolutions. In this respect no distinction can be made between stony and iron meteorites. However, a decisive difference will result from the fact that the more indestructible iron meteorites may traverse the asteroidal belt with impunity even if they do suffer collisions, while in such collisions stones will undergo further "grinding", and effectively start a new radiation life. This will give them a shorter average exposure age $T_{\rm S}$, connected with the long iron life $T_{\rm L}$ by

$$(1/T_{\rm S}) = (1/T_{\rm L}) + (1/T_{\rm C})$$
 (24)

where $T_{\rm C}$ stands for the life time with regard to collisions while traversing the belt. There our meteorites will spend, in a rough approximation, a fraction (r_0/R_0) of their lives in orbit. Thus

$$(1/T_{\rm C}) \approx (v_0/\lambda_i) \cdot (r_0/R_0)$$

= $(v_0/v) \cdot (r_0/R_0) \cdot (1/T_i)$. (25)

 T_i is the life time with regard to inelastic collisions of asteroidal bodies, derivied in (20) resp. (20 a). Since $T_{\rm L}$ is of the order of a few 10^8 years, a substantial shortening of the exposure age of stony meteorites is possible if $\gamma > 5/3$, say around 1.9. However, this would also lead to a small but perhaps detectable size dependence of the exposure ages.

⁹ T. Grjebine, Sciences 6, 45 [1965].

⁸ W. M. Alexander, C. W. McCracken, L. Sacretan, and O. E. Berg, Space Res. 3, 891 [1963].